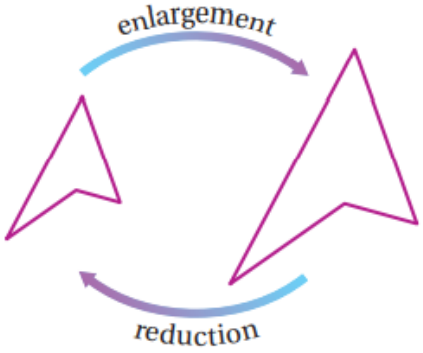
YEAR 10 MATHS 5.1: PROPERTIES OF GEOMETRICAL FIGURES

LESSON 1: SIMILARITY

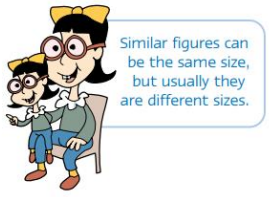
Congruent figures have identical (same)shape.

Similar figures have the same shape but not necessarily the same size.

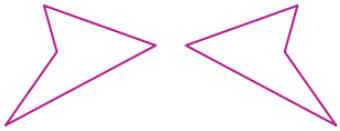
A photograph and its enlargement are similar figures.



A scale drawing is similar to the real-life shape.



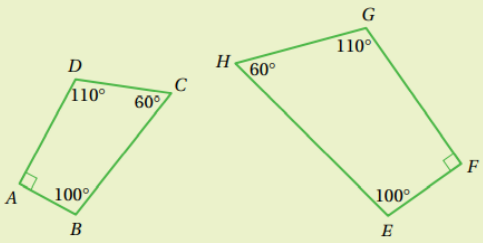
Change in **orientation (by rotation, refection Or translation)**

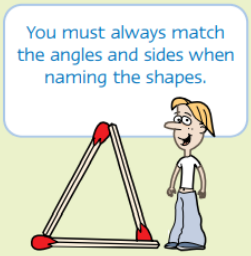


When naming similar figures, list the matching angles in the same order.

Examples

1 Name this pair of similar quadrilaterals correctly.

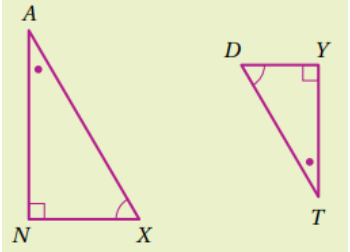




For this pair of similar quadrilaterals, the pairs of matching angles are $\angle A$ and $\angle F$, $\angle B$ and $\angle E$, $\angle C$ and $\angle H$, and $\angle D$ and $\angle G$.

$\therefore$ quadrilateral $ABCD$ ||| quadrilateral $FEHG$

2) List the pairs of matching sides and angles then write a similar figures statement.



Matching sides	Matching angles
AN and TY	$\angle A$ and $\angle T$
NX and YD	$\angle N$ and $\angle Y$
XA and DT	$\angle X$ and $\angle D$

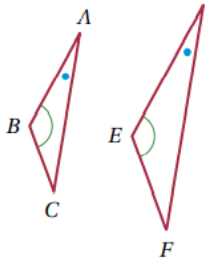
In this case, one correct similar figures statement is:

$\triangle ANX$ ||| $\triangle TYD$

EXERCISE 1:

1) Name each pair of similar figures correctly. Be careful with the order of the angles.

a



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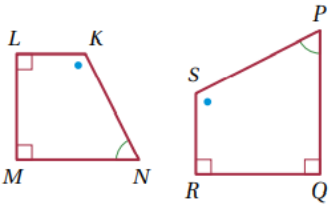

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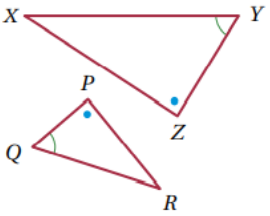

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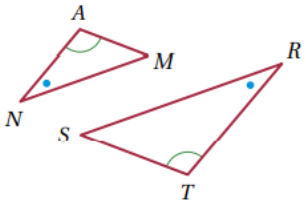

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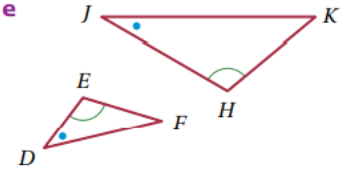

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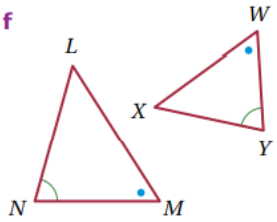
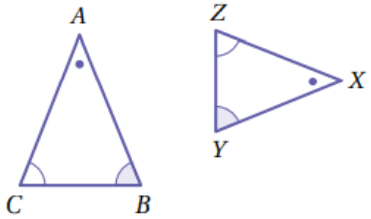


2) Draw each pair of similar figures in the same orientation, list the pairs of matching sides and angles and then write a similar figures statement.

a



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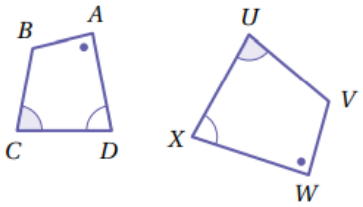
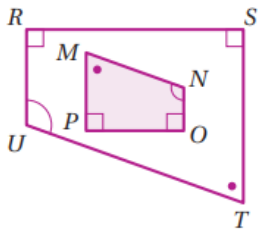


3) Write a similar figures statement for each pair of shapes:

a



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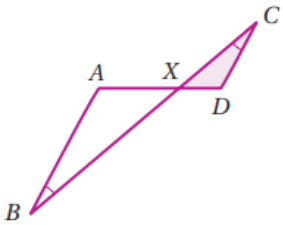

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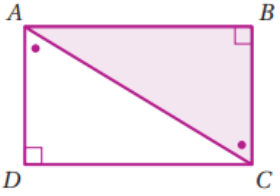
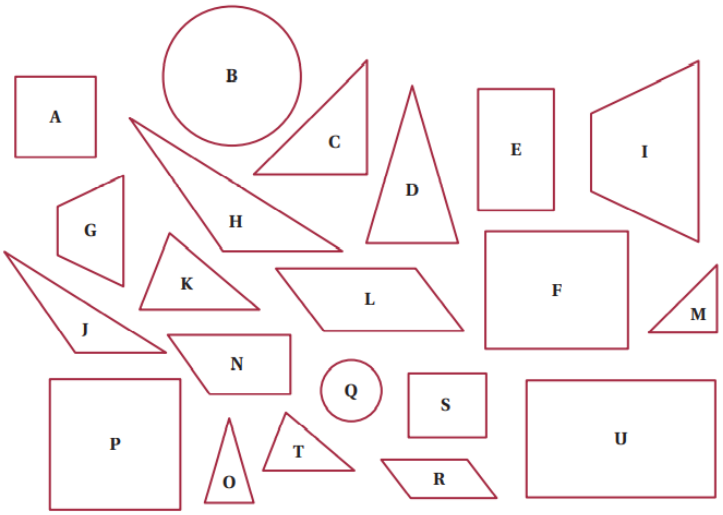


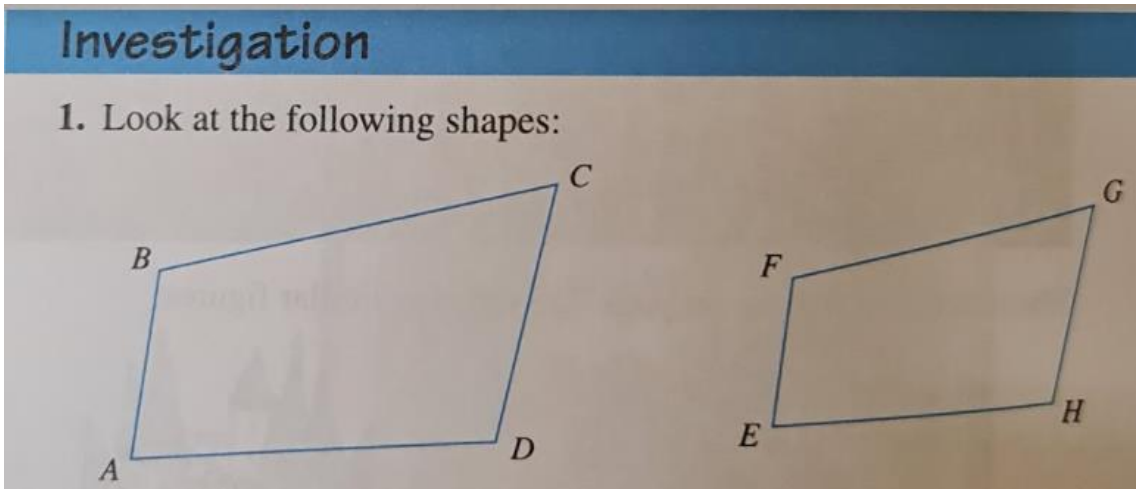
4) Match up pairs of similar figures to find out the odd one out!



LESSON 2: IDENTIFYING SIMILAR FIGURES

In similar figures:

- ✓ **All matching angles are equal.**
- ✓ **All matching sides are in the same ratio.**



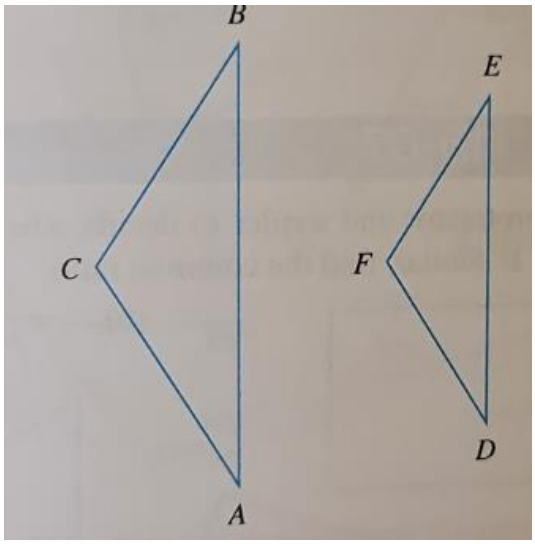
Using a protractor to measure the angles in each figure and complete the following tables.

Measure angles		**Matching pairs of angles are**:
<A=	<E=	
<B=	<F=	
<C=	<G=	
<D=	<H=	

Measure the lengths in milimetres		**Calculate ratio**
AB=	EF=	
BC=	FG=	
CD=	GH=	
DA=	HE=	

The ratio of the matching sides is _____

2)

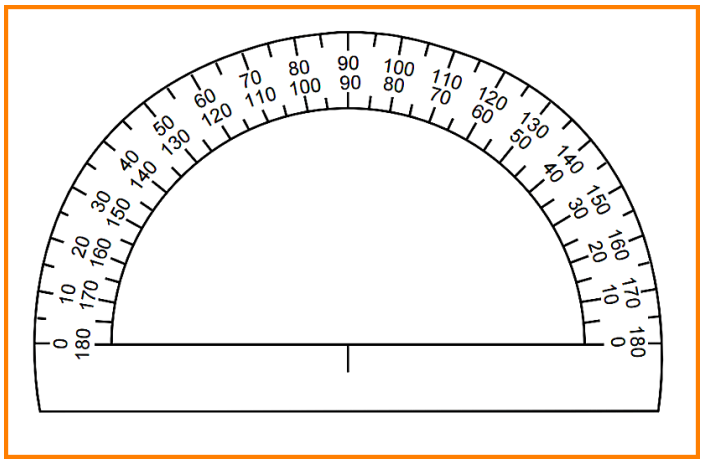


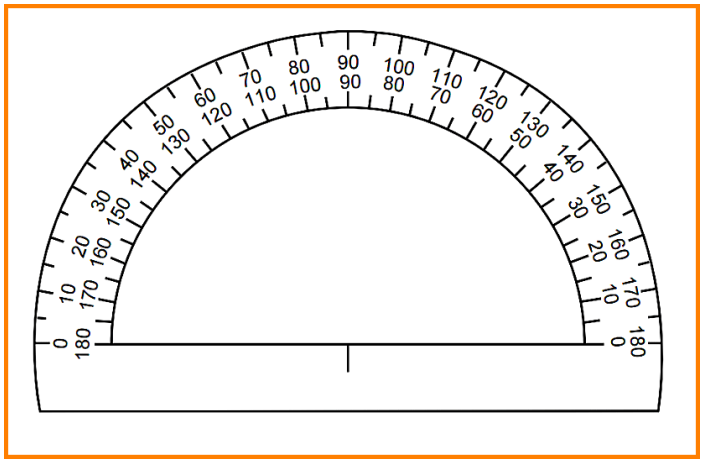
<A= <B= <C=	<D= <E= <F=	Matching pairs of angles are:

Measure the lengths in milimetres		**Calculate ratio**
AB=	**DE=**	
BC=	**EF=**	
CA=	**FD=**	

If you do not have a protractor, you can cut this shape out to use.

(Cut out the inner part so you can read the angle of the shapes.)





EXERCISE 2: Complete work in your book.

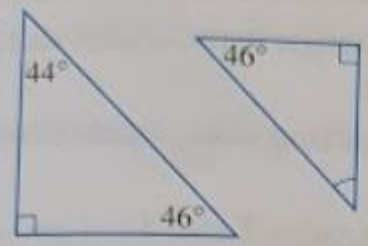
1. Use a protractor and a ruler to decide whether these pairs of figures are similar. If similar, find the common ratio.

(a)

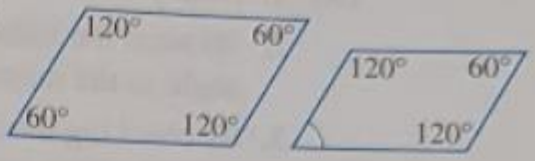
(b)

2. Find the size of the missing angles for each pair of similar figures:

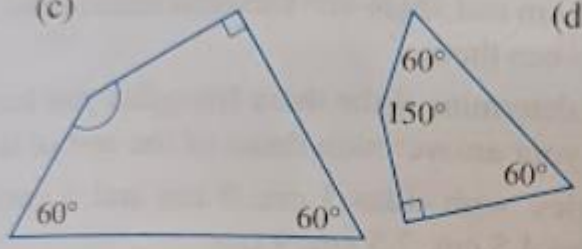
(a)



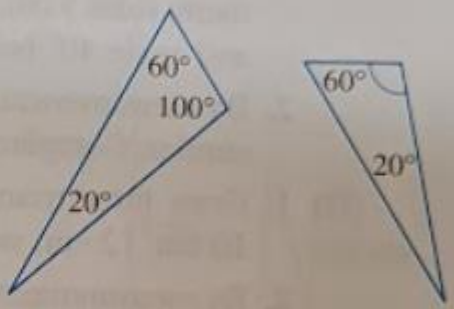
(b)



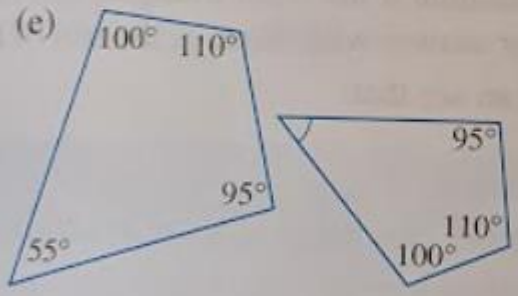
(c)



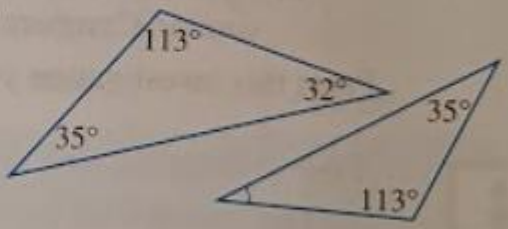
(d)



(e)



(f)



3. (a) Draw two squares that are similar. State the ratio of matching sides.
   (b) Draw two circles that are similar.
   (c) Draw two rectangles that are similar. State the ratio of matching sides.
   (d) Draw two right-angled triangles that are similar. State the ratio of matching sides.
   (e) Make up your own shape. Draw another that is similar to it.

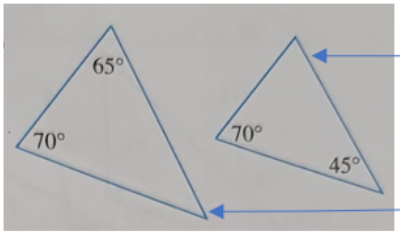
LESSON 3: SIMILAR TRIANGLES

Two Triangles are similar if

- **3 angles in each shape are equal.**
- **3 sides are in the same ratio.**
- **2 sides are in the same ratio and the angle in between the 2 sides is equal.**
  Note : The sum of 3 angles in a triangle adds up to 180^0.

Example

Are these triangles similar?



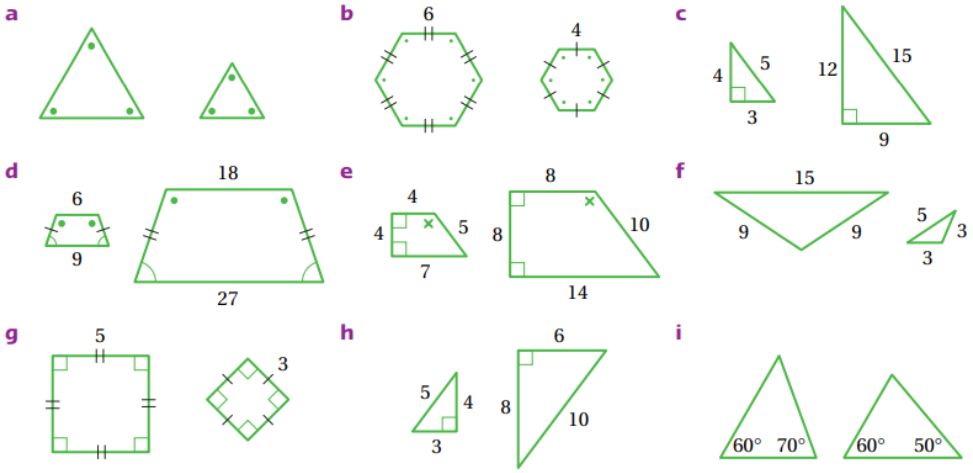
The missing angle in the first triangle = $180^0 - 70^0 - 65^0 = 45^0$

The missing angle in the second triangle = $180^0 - 70^0 - 45^0 = 65^0$

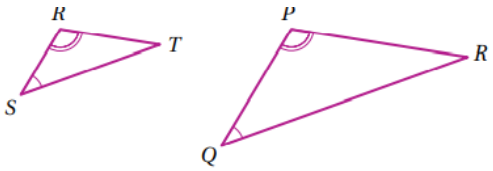
Yes they are, because three angles from one triangle equal to the three angles in the other triangle.

EXERCISE 3

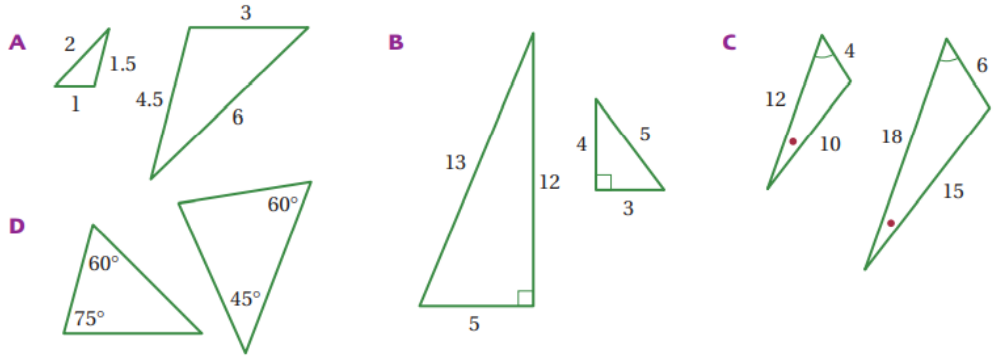
1 Decide whether or not each pair of figures are similar. Give reasons for your answers.
All lengths are in centimetres.



2 Is there enough information given here to show that these triangles are similar? Give reasons for your answer.



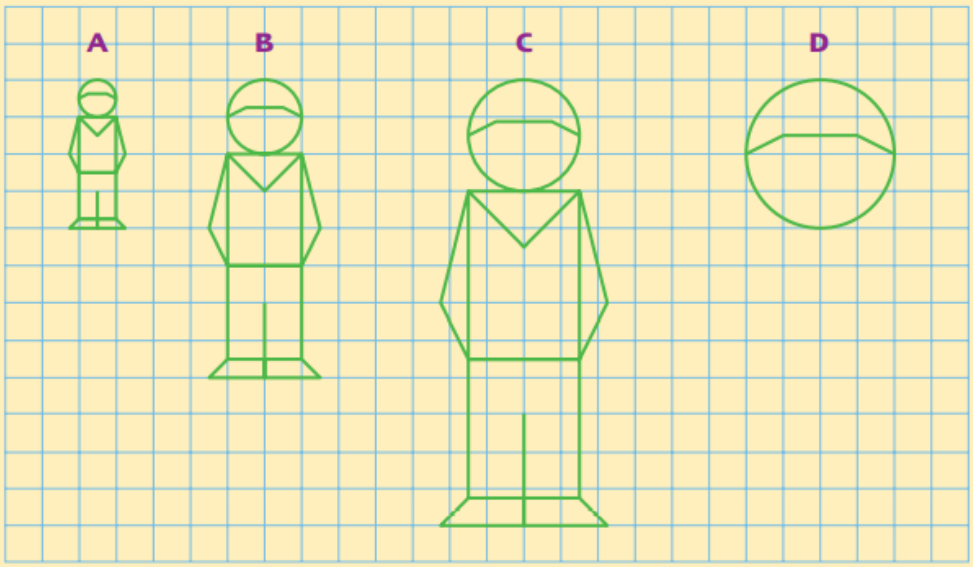
3 Which pair of triangles (**A**, **B**, **C** or **D**) are *not* similar? Give reasons for your answer.



Lesson 4 : Drawing similar figures

Construct an enlargement by using the same enlargement factor.

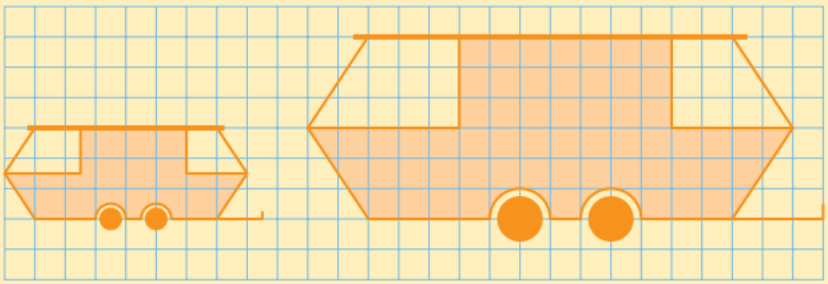
Drawing similar figures



- Copy and complete the table except for column **D**. Write your answers in units (length of 1 square = 1 unit).
- In column **D** write down what you think the measurements should be to make this figure similar to **A**, **B** and **C**.

Measurement	Figure			
	A	B	C	D
width of head				
width of shoulders				
length of legs				
height of figure				

Compare the overall length and height of each caravan.



- What can you say about the dimensions of the larger caravan compared with the smaller caravan?
- Are the drawings similar?

Extension: Draw an enlargement or reduction of a figure or object of your choice.

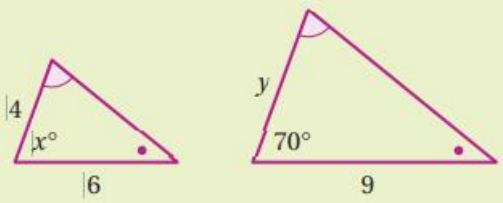
Lesson 5 : Finding unknown angles and sides of similar figures

If we know that a pair of figures are similar, we can use the properties of matching sides and angles to find unknown sides and angles.

Examples

For each pair of similar figures, find the values of the pronumerals.
Give reasons. All lengths are in centimetres.

1



In similar figures:
- All matching angles are equal.
- All matching sides are in the same ratio.

$x = 70$ (Matching angles in similar figures are equal.)

There are two methods of calculating the lengths of missing sides.

Method 1: Using the enlargement factor

In this case:

$$\text{Enlargement factor} = \frac{9}{6} \begin{matrix}\text{(side of large figure)} \\ \text{(side of small figure)}\end{matrix}$$

$$= 1\frac{1}{2}$$

Since y is in the large triangle, we *multiply*:

$y =$ (matching side in small figure) $\times$ (enlargement factor)

$y = 4 \times 1\frac{1}{2}$

$\therefore y = 6$

Method 2: Using the ratio of the matching sides

The unknown side (y) is in the large triangle, so we have to compare the sides in the large triangle with the matching sides in the small triangle.

Large triangle		Small triangle
y	is a matching side to	4
9	is a matching side to	6

Since the matching sides of similar figures are in the same ratio:

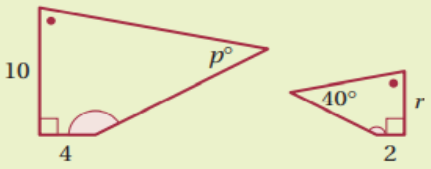
$$\frac{y}{4} = \frac{9}{6}$$
$$6y = 36$$
$$y = \frac{36}{6}$$
$$\therefore y = 6$$

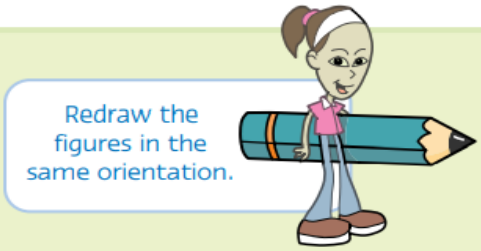
Cross-multiply:

$$\frac{y}{4} \times \frac{9}{6}$$

$6 \times y = 4 \times 9$

2

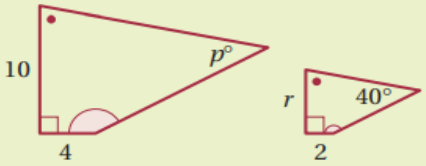




It is easier to match angles and sides when the similar figures have the same **orientation**.

$p = 40$ (Matching angles in similar figures are equal.)

To find the length of r:



Method 1

Enlargement factor $= \frac{4}{2}$

$= 2$

Since r is in the small similar figure, we *divide*:

$r = 10 \div 2$

$\therefore r = 5$

Method 2

$\frac{r}{10} = \frac{2}{4}$ (Matching sides of similar figures are in the same ratio.)

$4r = 20$ (Cross-multiply.)

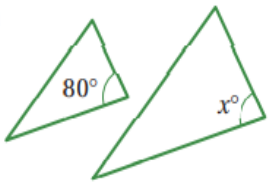
$r = \frac{20}{4}$

$\therefore r = 5$

Exercise 5:

1 For each pair of similar figures, find the value(s) of the variable(s). All lengths are in centimetres.

a



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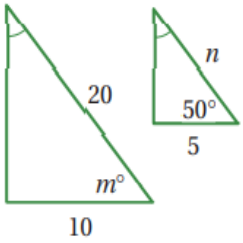

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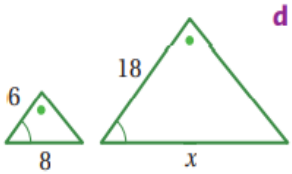

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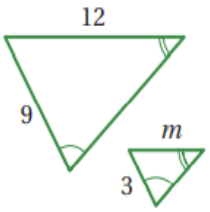

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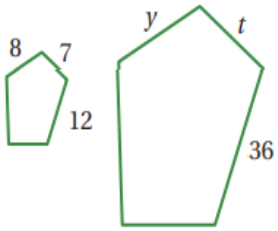

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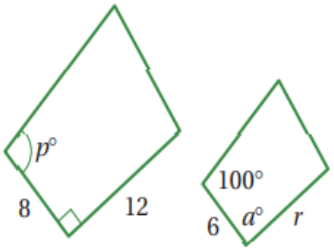

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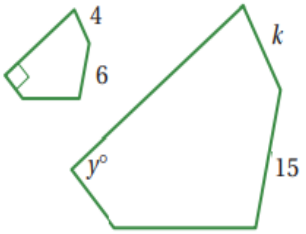
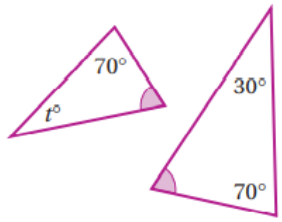


2 First redraw each pair of similar figures in the same orientation and then find the value of each variable. Give reasons. All lengths are in centimetres.

a



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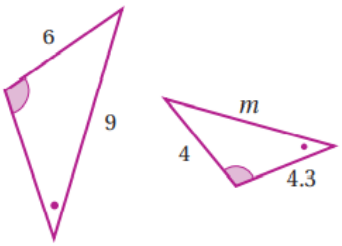

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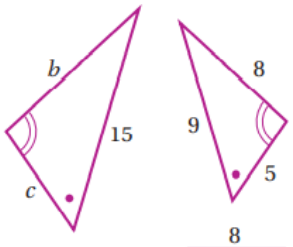

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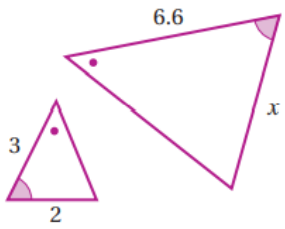

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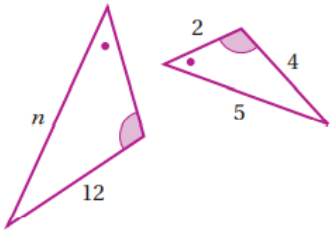

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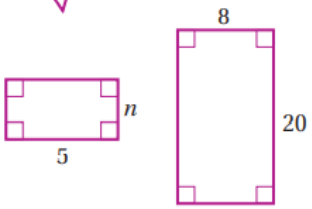
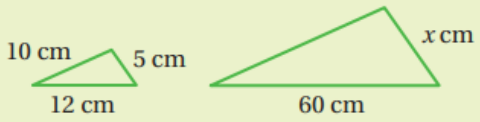


LESSON 6 : Solve problems involving similar figures.

Many everyday problems in design, architecture and engineering can be solved using the rules of similar figures.

Example

A triangle has sides 12 cm, 10 cm and 5 cm.
The longest side of a similar triangle is 60 cm.
What is the length of its shortest side?



Method 1

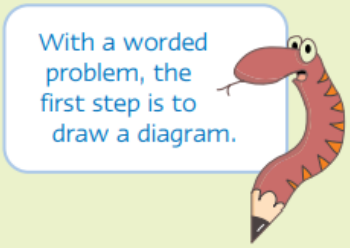
Enlargement factor $= \frac{60}{12}$

$= 5$

Since x is in the large triangle, we *multiply*.

$x = 5 \times 5$

$\therefore x = 25$



EXERCISE 6: complete these questions in your book.

1 A rectangle is 12 cm long and 4 cm wide. A similar rectangle is 24 cm long. What is its width?

2 A rectangle is 6 m long and 3 m wide. A similar rectangle is 15 m wide. What is its length?

3 A triangle has angles of 20°, 50° and 110°. What are the sizes of the angles in a similar triangle?

4 A triangle has sides 7 cm, 11 cm and 15 cm. The longest side of a similar triangle is 45 cm. What is the length of its shortest side?

5 A triangle has sides 24 cm, 28 cm and 40 cm. The longest side of a similar triangle is 5 cm. What is the length of its shortest side?

6 A triangle has sides 5 cm, 8 cm and 12 cm. The longest side of a similar triangle is 9 cm. What are the lengths of the other two sides?

7 The hypotenuse of a right-angled triangle is 40 mm. One of its other sides is 24 mm. The hypotenuse of similar triangle is 25 mm. What is the length of the side matching the 24 mm side in the first triangle?

8 A quadrilateral has sides 5 m, 7 m, 9 m and 12 m. The longest side of a similar quadrilateral is 12 cm. What is the length of its shortest side?

9 A quadrilateral has sides 60 cm, 47 cm, 35 cm and 30 cm. The longest side of a similar quadrilateral is 3 cm. Find the length of its shortest side.

10 A quadrilateral has angles 53°, 66°, 102° and 139°. A similar quadrilateral has sides twice as long. What is the size of its smallest angle?

11 The ratio of the lengths of the sides of two equilateral triangles is 7 : 3. The smaller triangle has sides 42 mm.

a Draw a diagram of the two triangles. **b** Why are they similar?

c Calculate the length of the sides of the larger triangle.

12 The ratio of the lengths of the sides of two isosceles triangles is 2 : 3. The larger triangle has side lengths 15 cm, 15 cm and 9 cm.

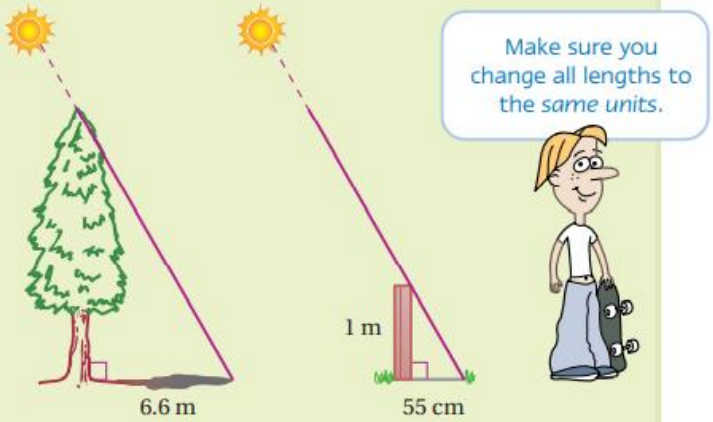
a Draw a diagram of the two triangles. **b** Are they similar?

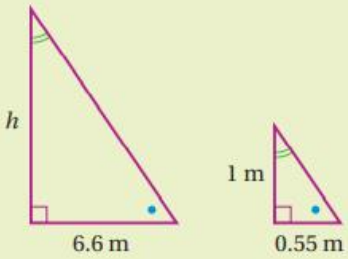
c Calculate the lengths of the sides of the smaller triangle.

LESSON 7: Find the lengths in the environment using Similar figures ratio.

Example

A tree casts a shadow 6.6 m long. At the same time a 1 m tall pole casts a shadow 55 cm long. How tall is the tree?





The two triangles are similar because the matching angles are equal.

Note that 55 cm = (55 ÷ 100) m = 0.55 m.

To find the height of the tree, h:

Method 1

$$\text{Enlargement factor} = \frac{6.6}{0.55}$$
$$= 6.6 \div 0.55$$
$$= 12$$

Since h is in the large triangle, we *multiply*:

$$h = 12 \times 1$$
$$\therefore h = 12$$

$\therefore$ the height of the tree is 12 m.

Check that this is a sensible answer.

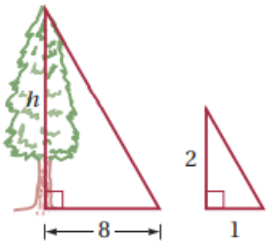
Method 2

$$\frac{h}{1} = \frac{6.6}{0.55}$$
$$h = 6.6 \div 0.55$$
$$\therefore h = 12$$

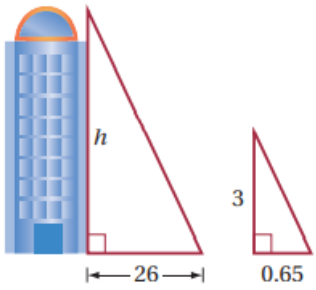
EXERCISE 7 : (note: you only need to choose 1 method in the example above to calculate the length)

21 Each diagram shows a tall object and a shadow stick casting shadows simultaneously. Use the diagrams to calculate the height (h) of each tall object. All lengths are in metres.

a



b



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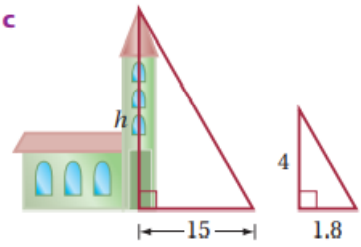
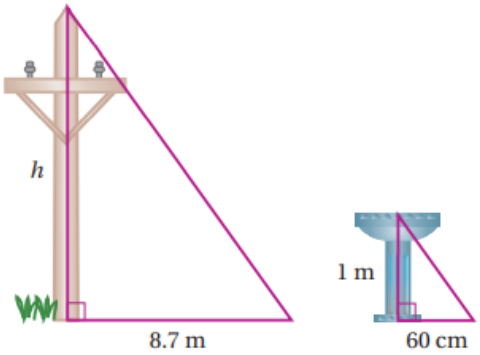


22 A telegraph pole casts a shadow 8.7 m long. At the same time a birdbath casts a shadow 60 cm long. How tall is the pole?



23 A building casts a shadow 10.4 m long. At the same time a metre rule casts a shadow 40 cm long.

a Draw a diagram showing this information.

b How tall is the building?

24 The shadow from a flagpole is 17.5 m long. At the same time, the shadow from a 2 m high post is 1.4 m long.

a Draw a diagram showing this information.

b What is the height of the flagpole?

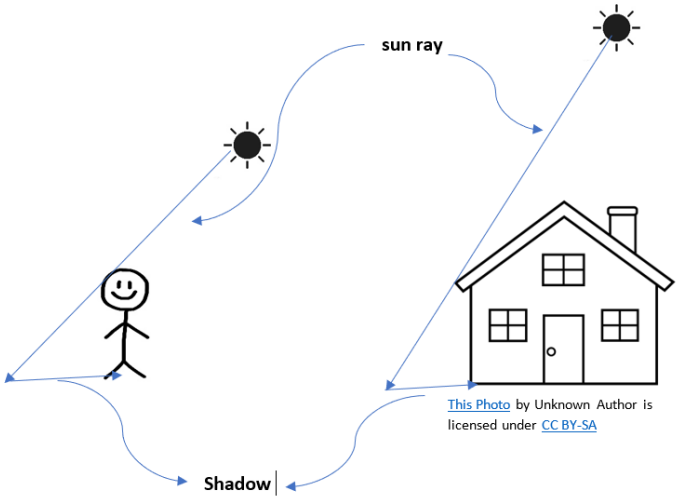
25 A light pole 6 m tall casts a shadow 2.5 m long on level ground. At the same time, a building casts a shadow 15 m long.

a Draw a diagram showing this information.

b How much taller than the pole is the building?

Practical activity:

Find the height of a tree near your house or the height of your house.



Use the steps below to calculate the height of the house or the tree.

Step 1) Measure the shadows of yourself and the house and label them on the diagram.

Step 2) Measure your height and label on the diagram.

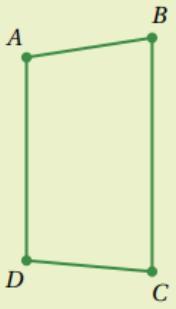
Step 3) Use your knowledge of similar figure Ratio to calculate the height of the house.

Lesson 8 : Enlarging or reducing figures

Similar figures can be drawn by enlarging (or reducing) an existing figure. If a figure is enlarged, the enlargement factor or scale factor used is greater than 1. If a figure is reduced, the scale factor is less than 1.

Examples

1 Enlarge quadrilateral $ABCD$ using point O as the centre of enlargement and a scale factor of 2.

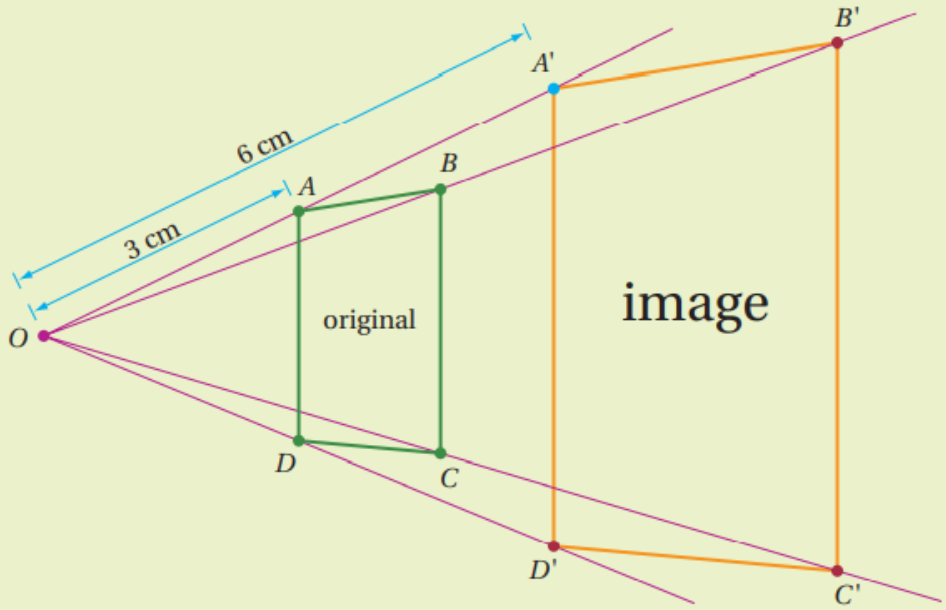


Step 1 Draw lines from the point O through each vertex of the original figure.

Step 2 Measure the distance from point O to point A and multiply this length by the scale factor (in this case 2). Mark this new distance from O along the line and label the image A'.

Step 3 Follow the same procedure for points B, C and D, labelling the images of these points B', C' and D'.

Step 4 Join points A', B', C' and D' to form the enlargement.



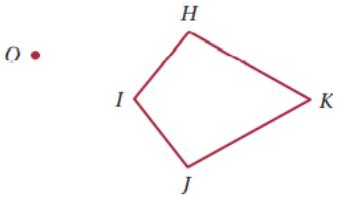
Use a ruler to compare the lengths of the matching sides. In this case the scale factor is 2, so the lengths of the matching sides in the image $A'B'C'D'$ should be *twice* the length of those in the original, $ABCD$. So $ABCD$ and $A'B'C'D'$ are similar figures (the same shape but different sizes).

Exercise 8

Copy each figure and point O, the centre of enlargement. Construct an enlargement using the given enlargement factor.

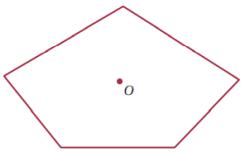
a) scale factor=2

b enlargement factor 3



c)

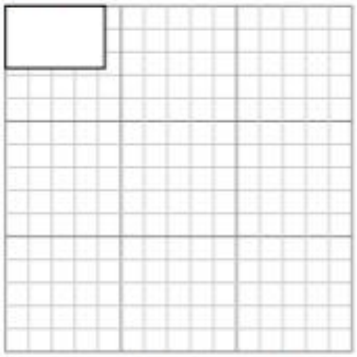
enlargement factor 3



LESSON 9 : Revision Practice drawing (by using scale factor)

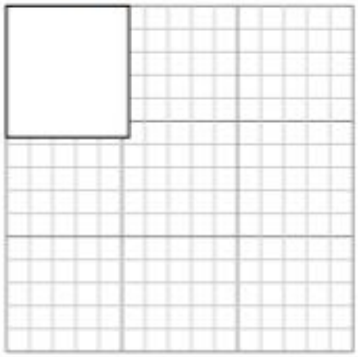
Draw each rectangle to the scale shown and determine the new dimensions.

1) The rectangle below has the dimensions: 4.3 × 2.7



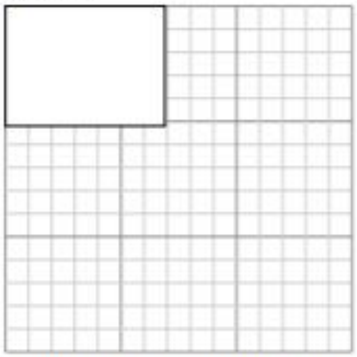
Create another rectangle that is scaled to 9 times the size of the current rectangle.

2) The rectangle below has the dimensions: 5.3 × 5.7



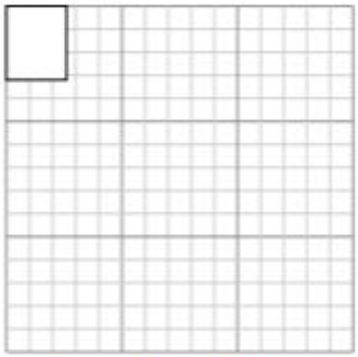
Create another rectangle that is scaled to 4 times the size of the current rectangle.

3) The rectangle below has the dimensions: 6.9 × 5.2



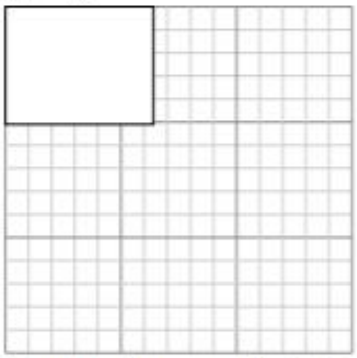
Create another rectangle that is scaled to 4 times the size of the current rectangle.

4) The rectangle below has the dimensions: 2.6 × 3.2



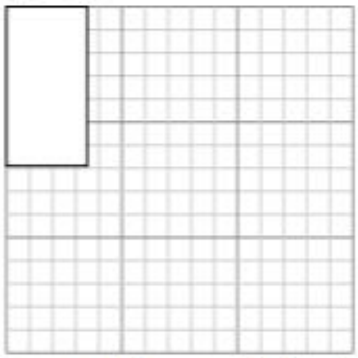
Create another rectangle that is scaled to 9 times the size of the current rectangle.

5) The rectangle below has the dimensions: 6.4 × 5.1



Create another rectangle that is scaled to 4 times the size of the current rectangle.

6) The rectangle below has the dimensions: 3.5 × 6.9



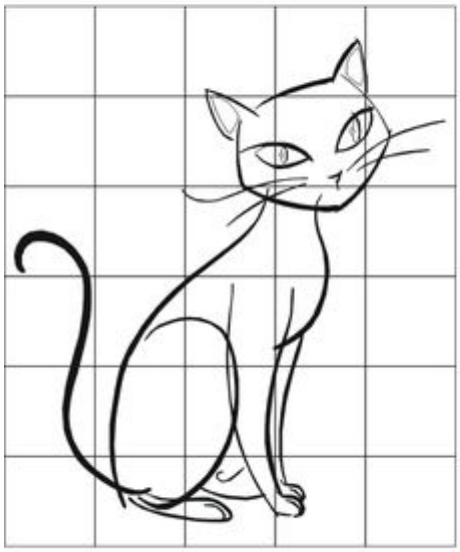
Create another rectangle that is scaled to 4 times the size of the current rectangle.

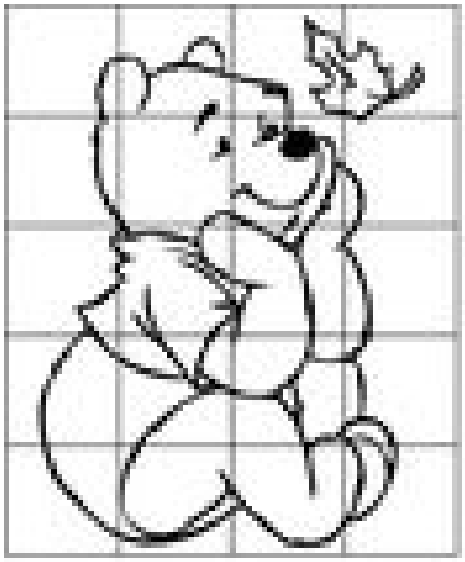
12.9 x 8.1

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Use the grid to help you with drawing.

NAME: _____
AGE: _____
DATE: _____





NAME: _____
AGE: _____
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