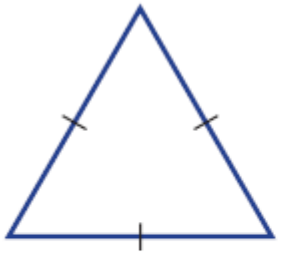
Right Angled Triangles

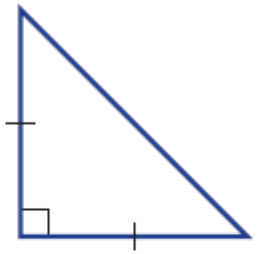
REVISION

1 Which of the following triangles contain a side known as the hypotenuse?

a



b



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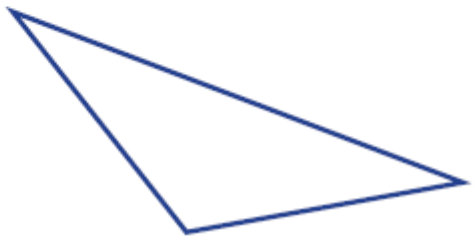
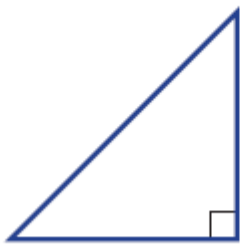


Only right-angled triangles have a hypotenuse.

2 Copy these triangles into your workbook and label the hypotenuse.

a

b



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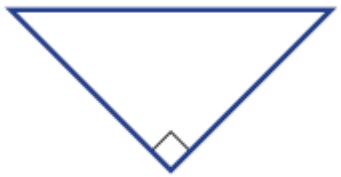
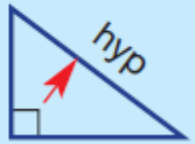
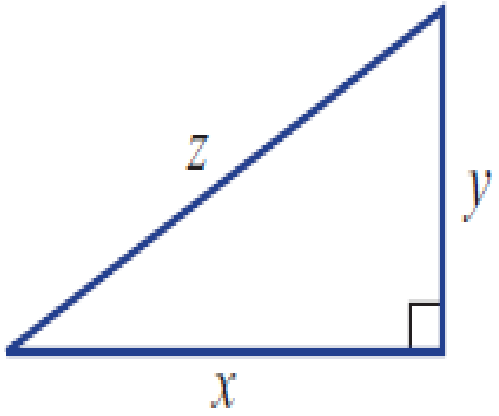


Draw an arrow across from the right angle to find the hypotenuse (hyp).



3 Write the relationship between the sides of these triangles.

a



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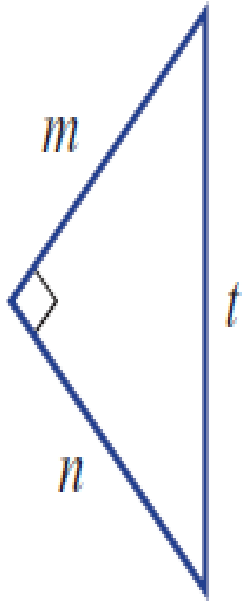

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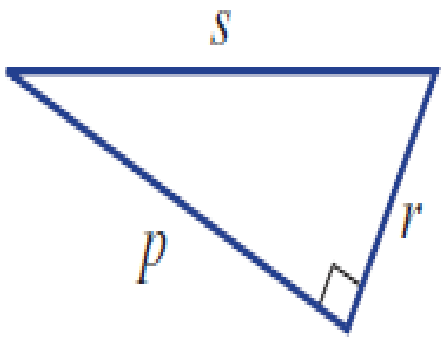
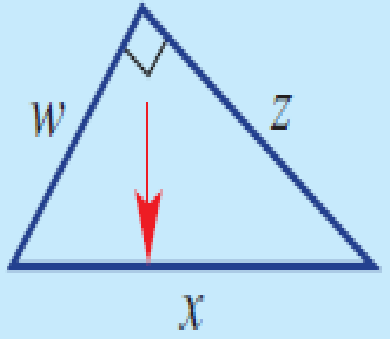




$x^2 = w^2 + z^2$

4 Find the value of $a^2 + b^2$ if:

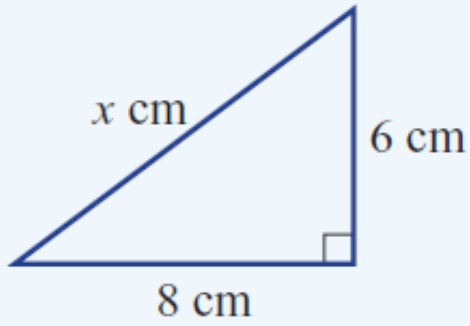
a $a = 3$ and $b = 4$

b $a = 3$ and $b = 5$

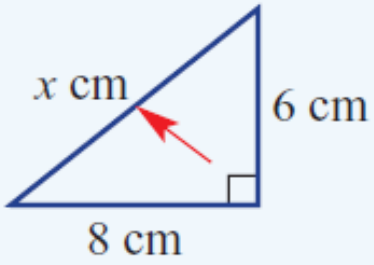
c $a = 3$ and $b = 6$

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse of the triangle shown.

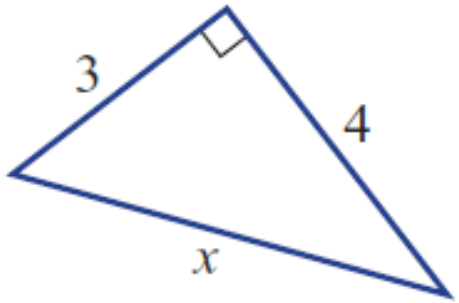


SOLUTION	EXPLANATION
$x^2 = 6^2 + 8^2$	Write the relationship for the given triangle using Pythagoras' theorem.
$= 36 + 64$	
$= 100$	
$x = \sqrt{100}$	Take the square root to find x.
$= 10$	
$\therefore$ Hypotenuse length = 10 cm.	Write your answer.



5 Find the length of the hypotenuse in these right-angled triangles.

a



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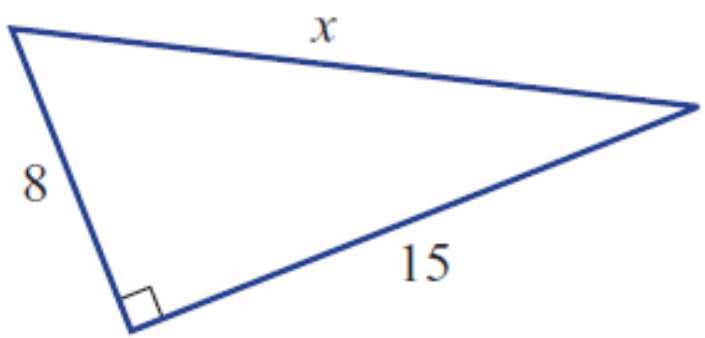

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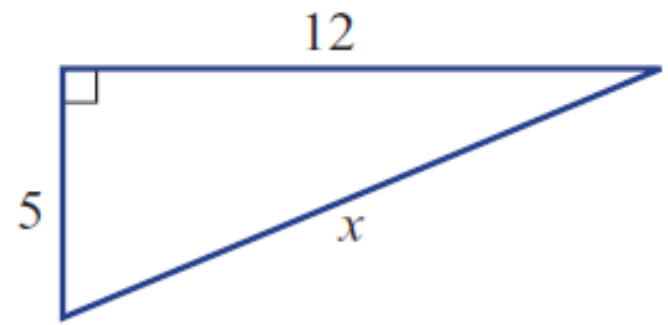

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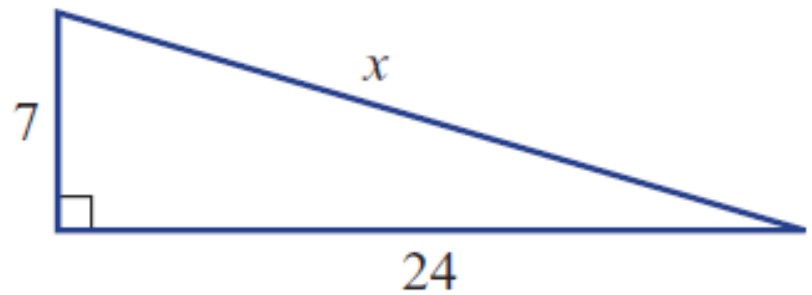

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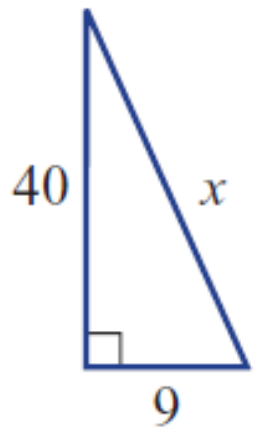

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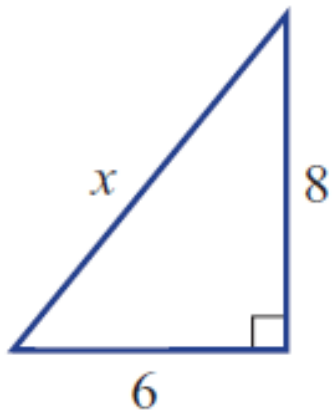
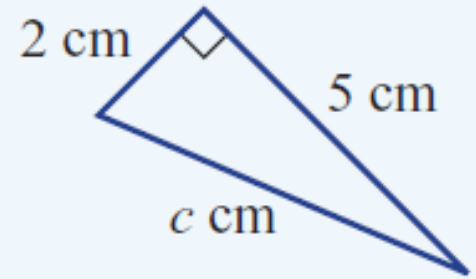
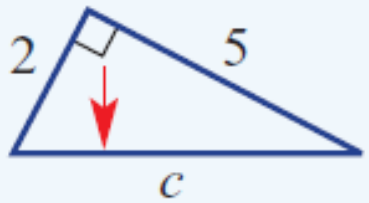


Example 2 Finding the length of the hypotenuse as a decimal

Find the length of the hypotenuse in this triangle, correct to 1 decimal place.

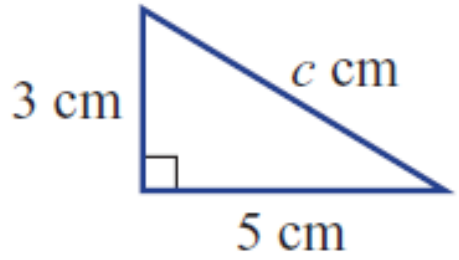


SOLUTION	EXPLANATION
$c^2 = 5^2 + 2^2$	Write the relationship for this triangle, where c is the length of the hypotenuse.
$= 25 + 4$	
$= 29$	Simplify.
$c = \sqrt{29}$	Take the square root to find c.
$c = 5.38516\ldots$	
$c = 5.4$	Round 5.3⑧516 … to 1 decimal place.
$\therefore$ Hypotenuse length = 5.4 cm.	



6 Find the length of the hypotenuse in these triangles, correct to 1 decimal place.

a



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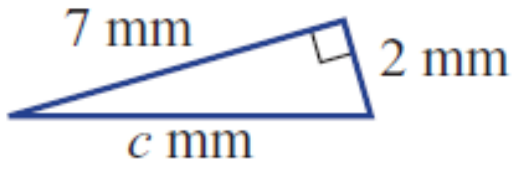

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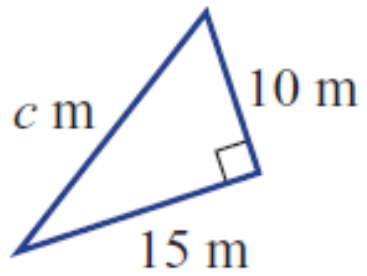
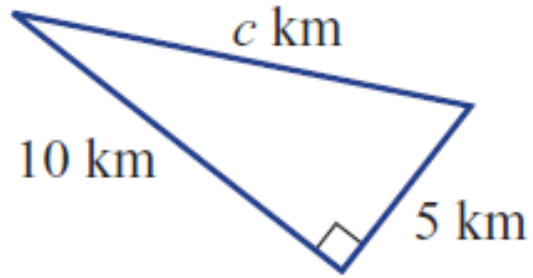


If $c^2 = 34$, then $c = \sqrt{34}$. Use a calculator to find the decimal.

d



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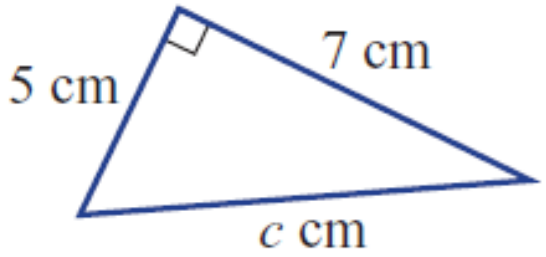

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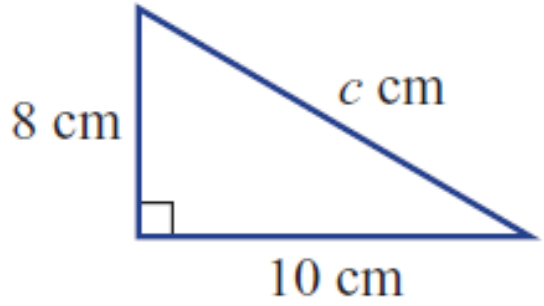
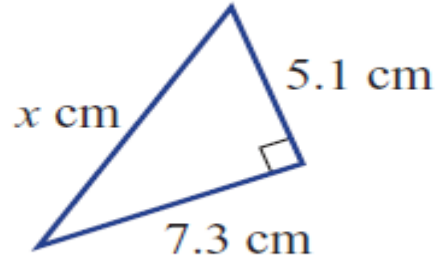


7 Find the value of the hypotenuse in these triangles, correct to 2 decimal places.

a



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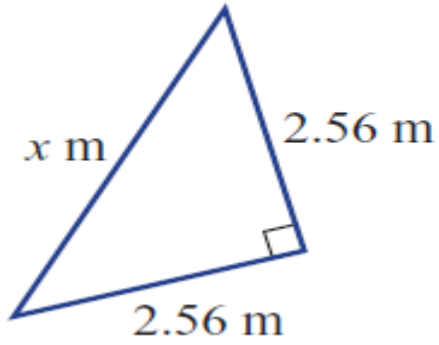

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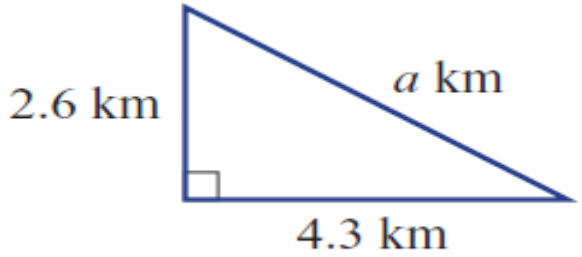

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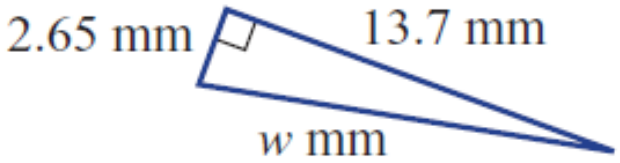

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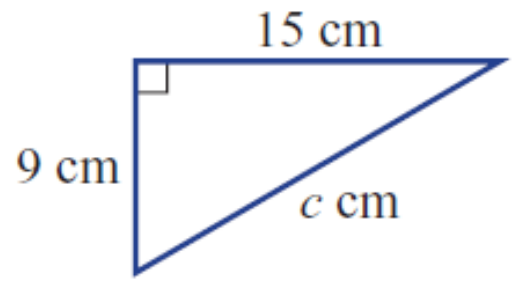

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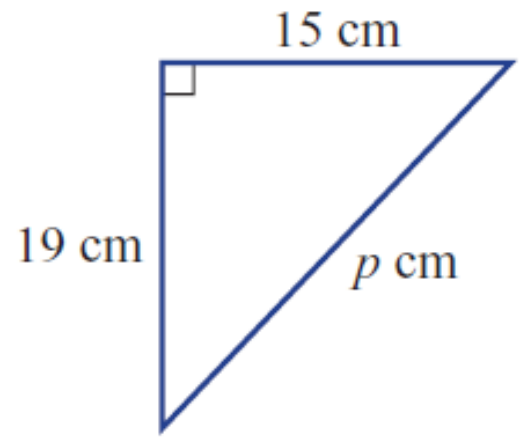

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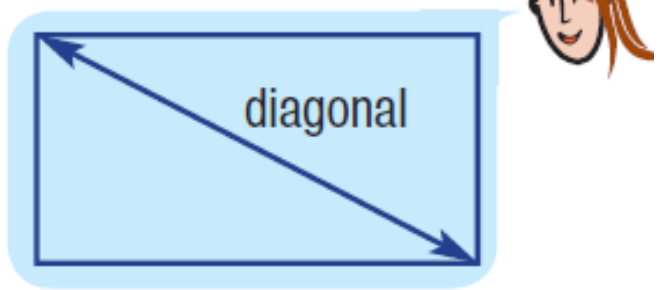
8 A LCD plasma TV is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to 1 decimal place.

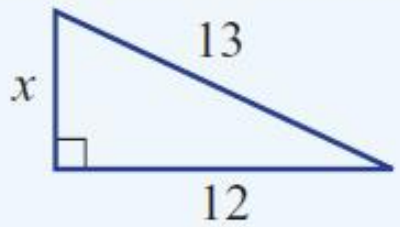


Finding the Length of the Shorter Sides

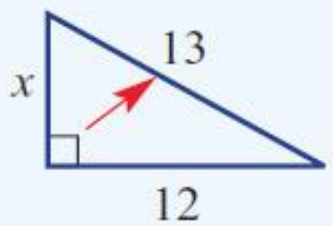
Using Pythagoras' theorem, we can determine the length of the shorter sides of a right-angled triangle. The angled support beams on a rollercoaster ride, for example, create right-angled triangles with the ground. The vertical and horizontal distances are the shorter sides of the triangle.

Example 3 Calculating a shorter side

Determine the value of x in the triangle shown, using Pythagoras' theorem.

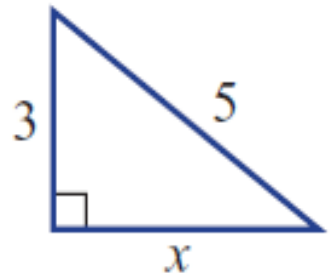


SOLUTION	EXPLANATION
$13^2 = x^2 + 12^2$	Write the relationship for this triangle using Pythagoras' theorem with 13 as the hypotenuse.
$x^2 + 12^2 = 13^2$	
$x^2 = 13^2 - 12^2$	Rewrite the rule with the x^2 on the left-hand side.
$= 169 - 144$	
$= 25$	Simplify.
$x = \sqrt{25}$	Take the square root to find x.
$\therefore x = 5$	



4 Determine the value of x in these triangles, using Pythagoras' theorem.

a



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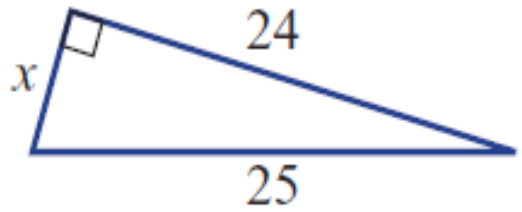

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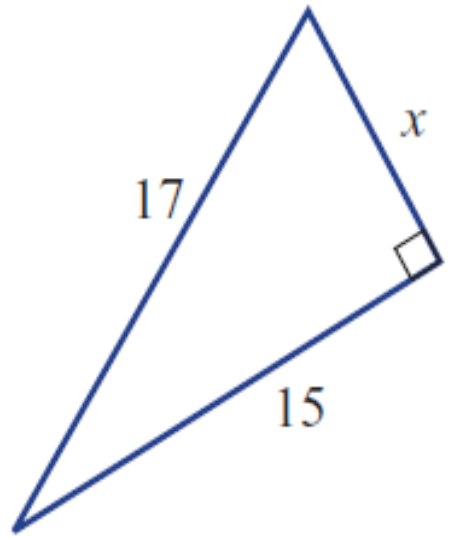
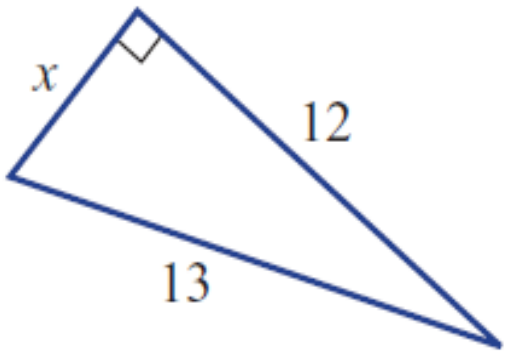


In $c^2 = a^2 + b^2$, c is always the hypotenuse.

d



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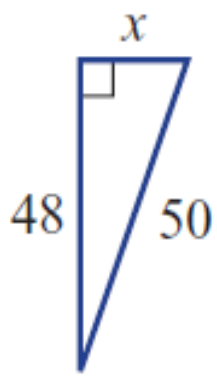

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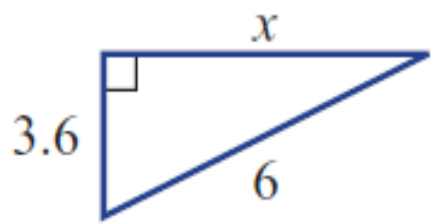
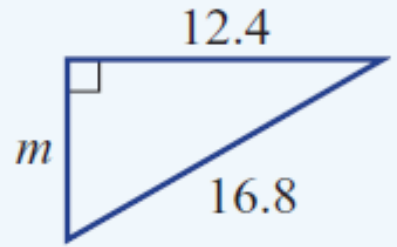


Example 4 Finding a shorter side length as a decimal value

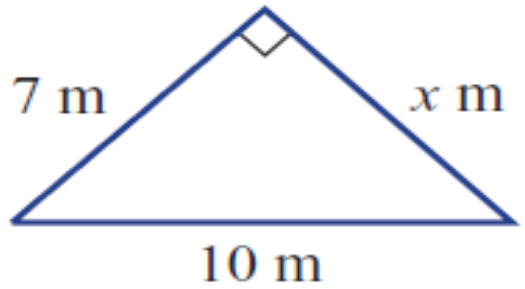
Determine the value of m in the triangle, correct to 1 decimal place.



SOLUTION	EXPLANATION
$16.8^2 = m^2 + 12.4^2$	Write the relationship for this triangle.
$m^2 + 12.4^2 = 16.8^2$	
$m^2 = 16.8^2 - 12.4^2$	Make m^2 the subject by first swapping the LHS and RHS.
$= 128.48$	Simplify, using your calculator.
$m = \sqrt{128.48}$	Take the square root of both sides to find m.
$= 11.3349\ldots$	
$m = 11.3$	Round your answer to 1 decimal place.

5 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to 1 decimal place.

a



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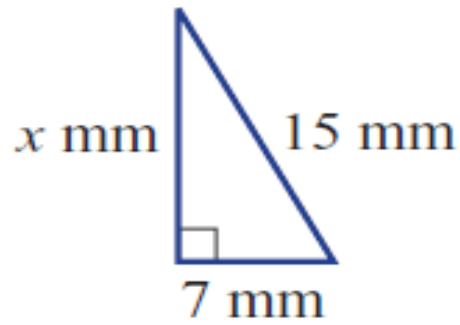

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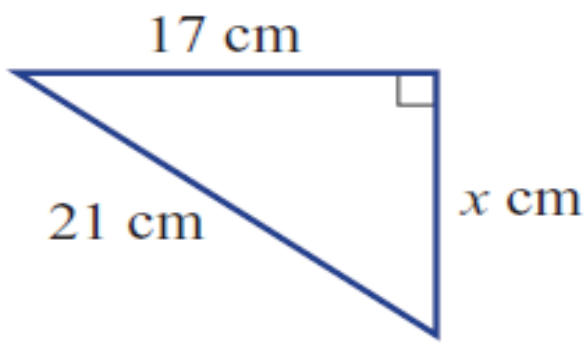

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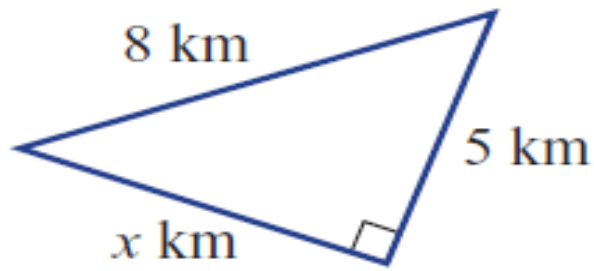

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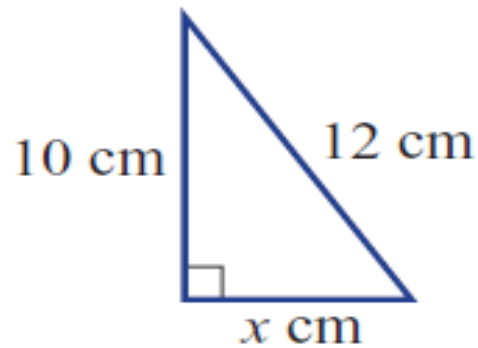

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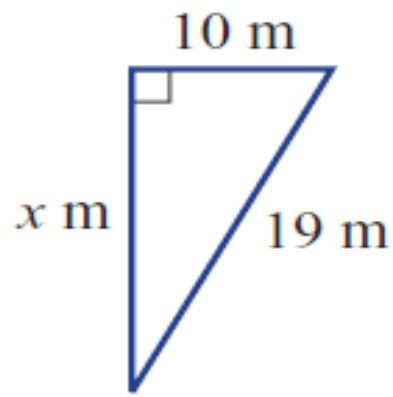
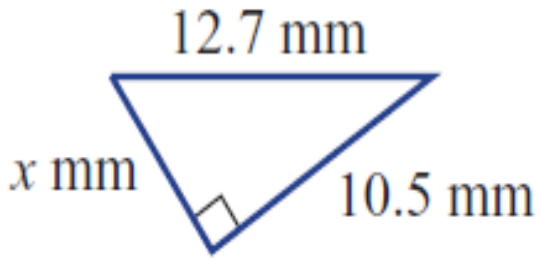


Lesson 3

6 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to 2 decimal places.

a



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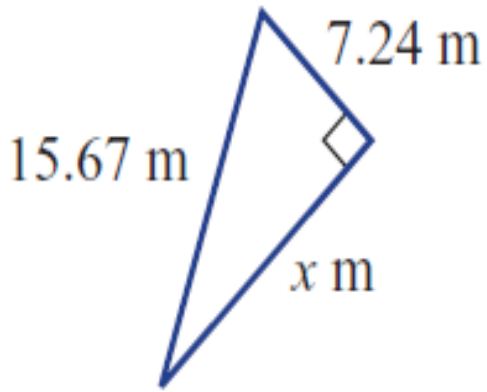

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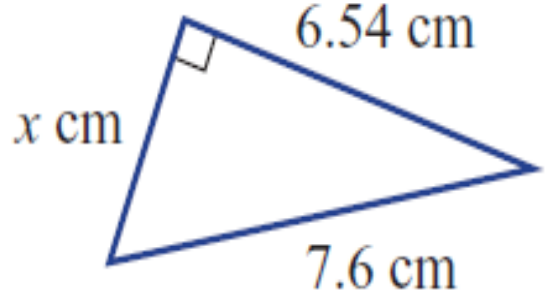

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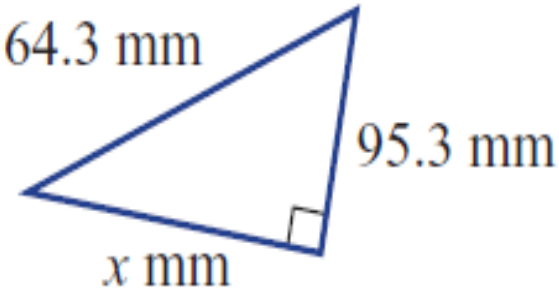

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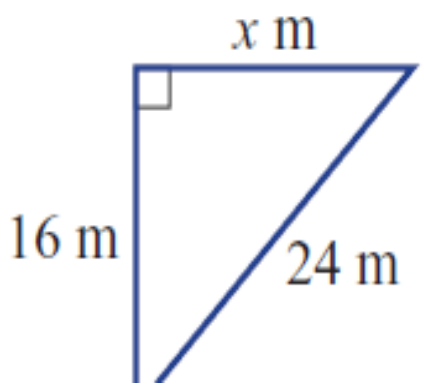
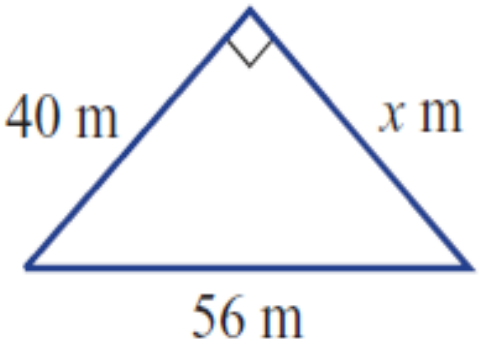
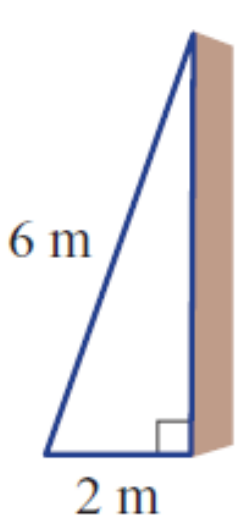


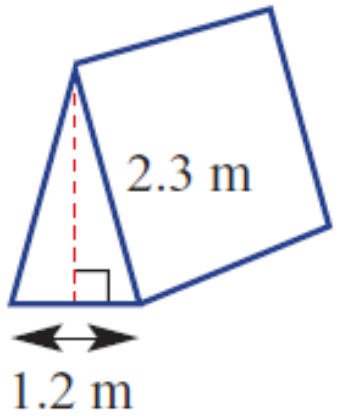
f



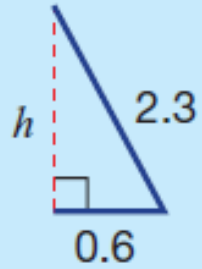
7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to 2 decimal places.



8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to 1 decimal place.



Identify the right-angled triangle.

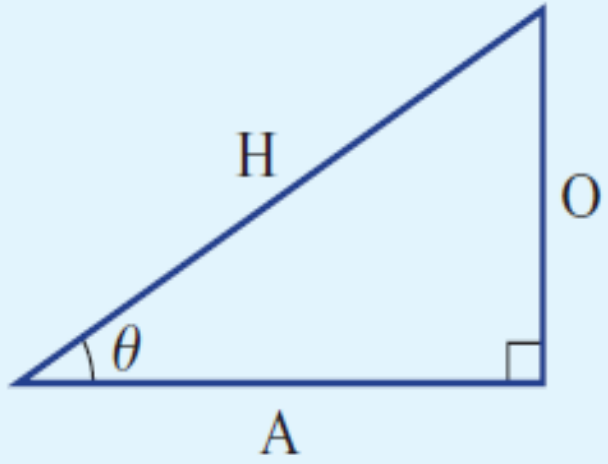


Trigonometric Ratios

- When working with right-angled triangles, label each side of the triangle O (opposite), A (adjacent) and H (hypotenuse).
- The three trigonometric ratios are:

$$\sin\theta = \frac{O}{H} \qquad \cos\theta = \frac{A}{H} \qquad \tan\theta = \frac{O}{A}$$

We can remember this as **SOH CAH TOA**.

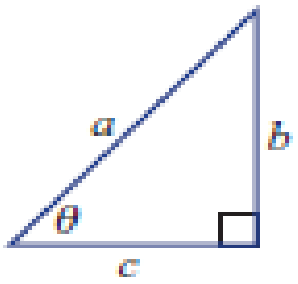


https://www.youtube.com/watch?v=zU94BB9aUgE

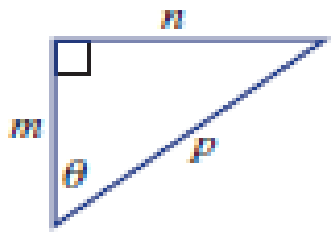
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

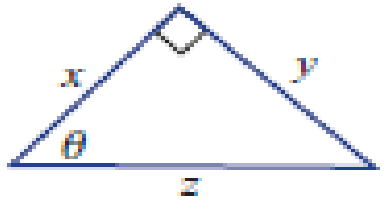
a



b

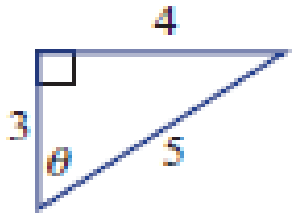


c

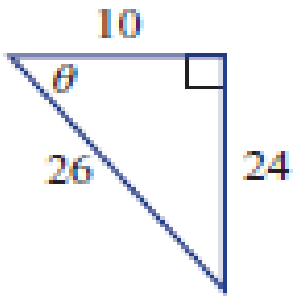


Use SOH CAH TOA after labelling the sides as O, A and H.

d



e



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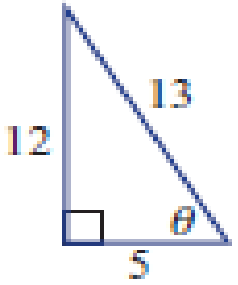
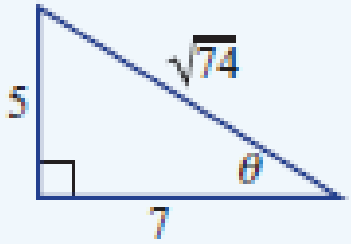


Example 9 Writing a trigonometric ratio

Write down the ratio of cos θ for this triangle.



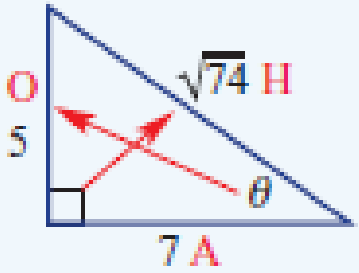
SOLUTION

$$\cos\theta = \frac{\text{A}}{\text{H}}$$

$$\cos\theta = \frac{7}{\sqrt{74}}$$

EXPLANATION

Label the sides of the triangle.

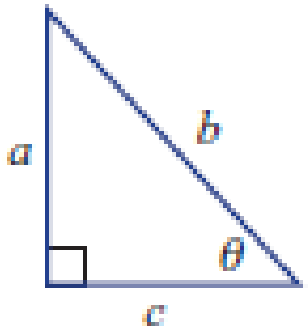


SOH CAH TOA tells us cos θ is $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Substitute the values for the adjacent (A) and hypotenuse (H).

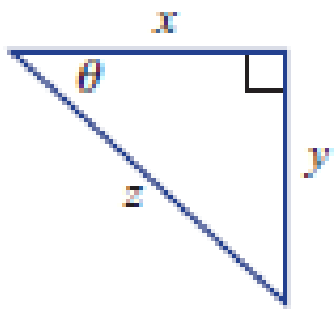
6 Write the trigonometric ratio asked for in each of the following.

a



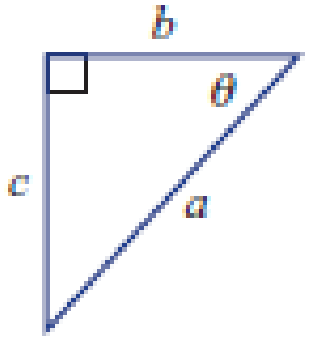
$\tan\theta =$

b



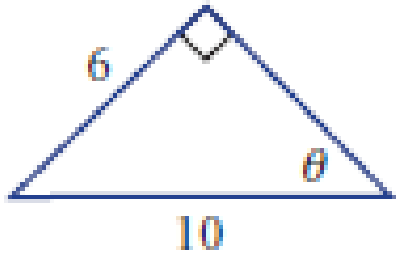
$\sin\theta =$

c



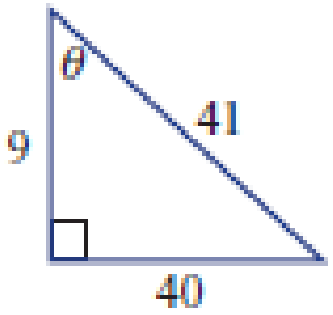
$\cos\theta =$

d



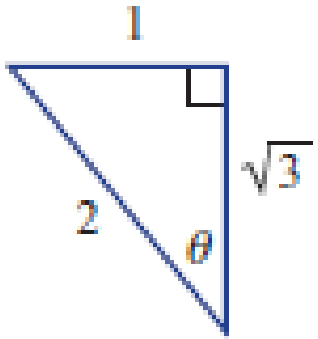
$\sin\theta =$

e



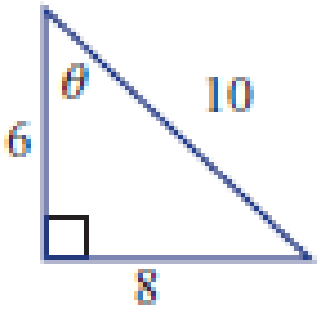
$\sin\theta =$

f



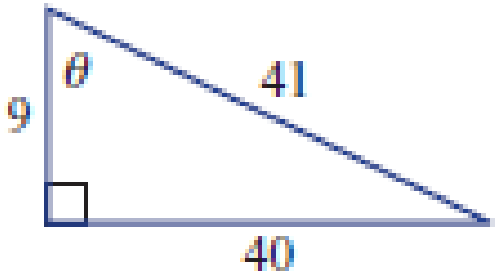
$\tan\theta =$

g



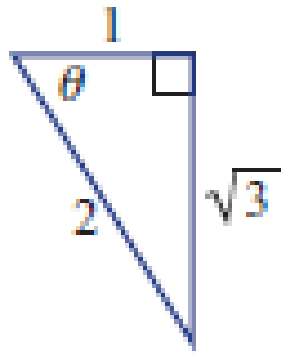
$\tan\theta =$

h



$\cos\theta =$

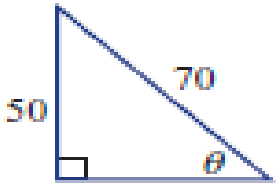
i



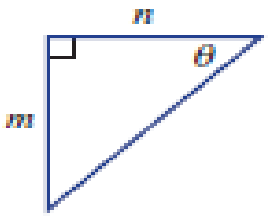
$\tan\theta =$

9 For each of the triangles below, decide which trigonometric ratio (i.e. sin, cos or tan) you would use.

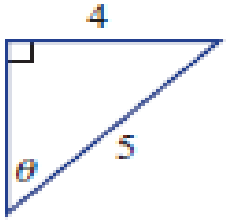
a



b

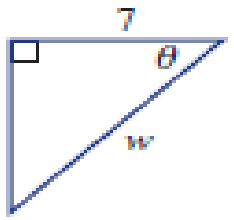


c

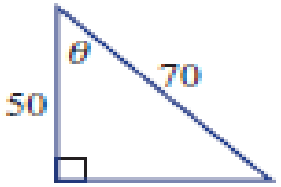


First decide which two sides you have: *O*, *A* or *H*?

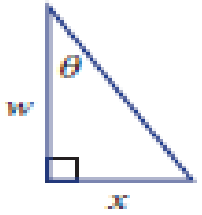
d



e



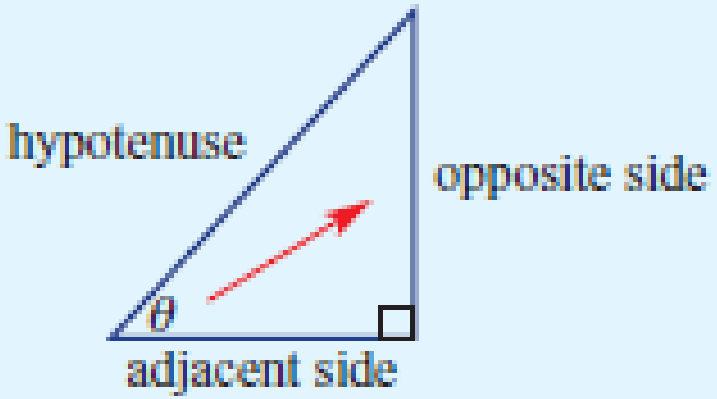
f

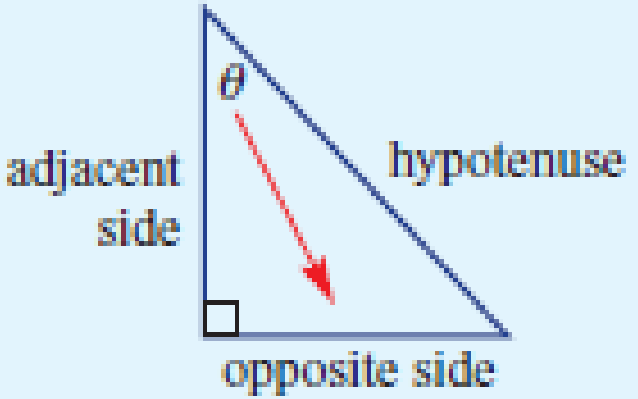


Lesson 5

Finding Unknown Sides

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The hypotenuse is always opposite the right angle.
  - The *adjacent* side is next to the **angle of reference** (θ).
  - The *opposite* side is opposite the angle of reference.





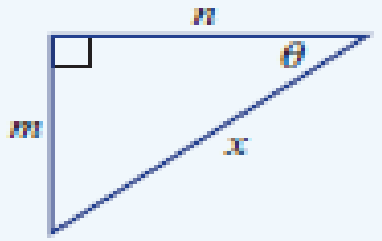
Angle of reference The angle in a right-angled triangle that is used to determine the opposite side and the adjacent side

https://www.youtube.com/watch?v=a5WQlcFTXyk&t=164s

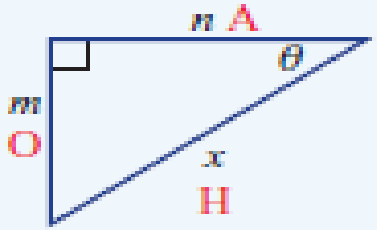
Example 8 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



SOLUTION



a $\sin\theta = \frac{m}{x}$

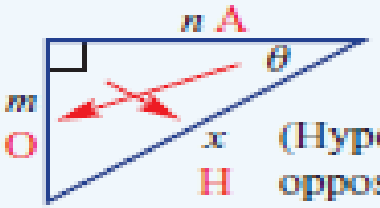
b $\cos\theta = \frac{n}{x}$

c $\tan\theta = \frac{m}{n}$

EXPLANATION

Use arrows to label the sides correctly.

(opposite angle θ)



(Hypotenuse is always opposite the right angle.)

SOH CAH TOA

$$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{m}{x}$$

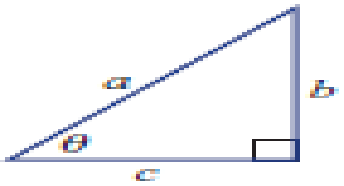
$$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{n}{x}$$

$$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{m}{n}$$

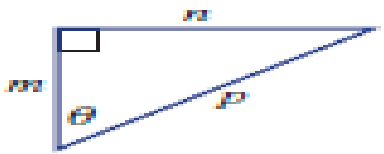
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

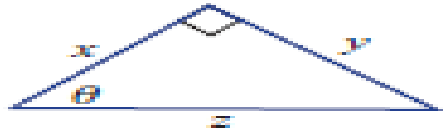
a



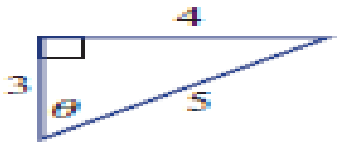
b



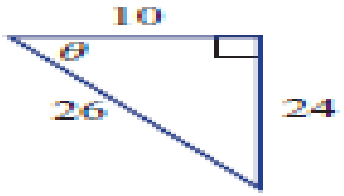
c



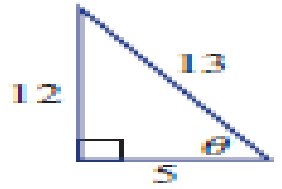
d



e

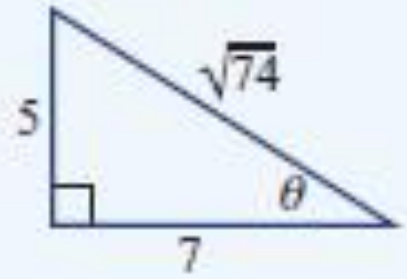


f



Example 9 Writing a trigonometric ratio

Write down the ratio of $\cos\theta$ for this triangle.



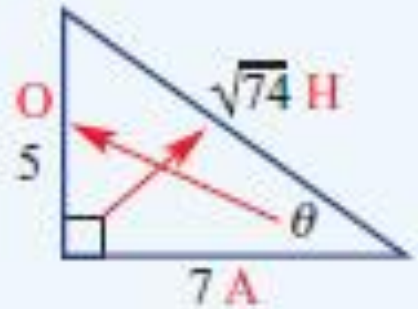
SOLUTION

$$\cos\theta = \frac{\text{A}}{\text{H}}$$

$$\cos\theta = \frac{7}{\sqrt{74}}$$

EXPLANATION

Label the sides of the triangle.

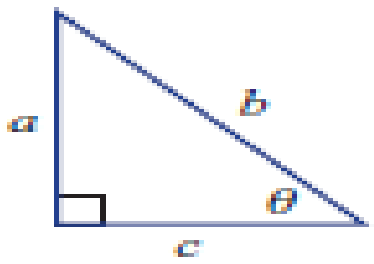


SOH CAH TOA tells us $\cos\theta$ is $\dfrac{\text{adjacent}}{\text{hypotenuse}}$.

Substitute the values for the adjacent (A) and hypotenuse (H).

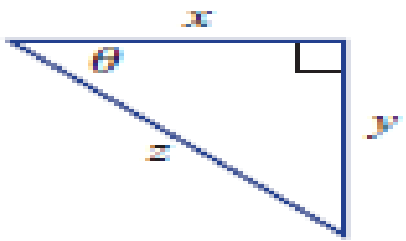
6 Write the trigonometric ratio asked for in each of the following.

a



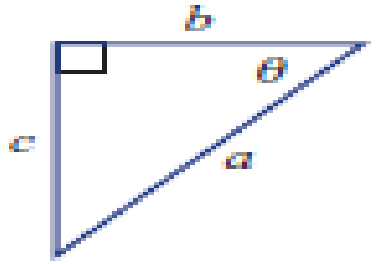
$\tan\theta =$

b



$\sin\theta =$

c

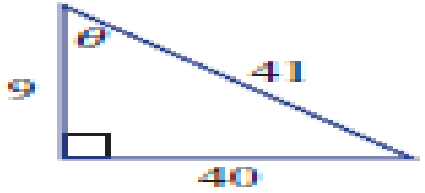


$\cos\theta =$

d

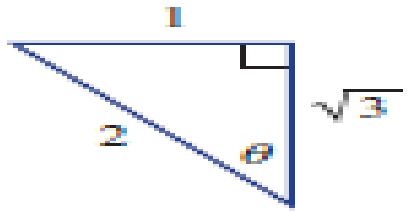
$\sin\theta =$

e



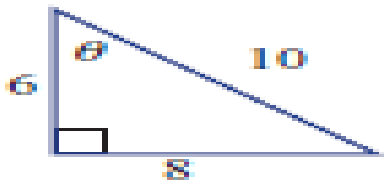
$\sin\theta =$

f



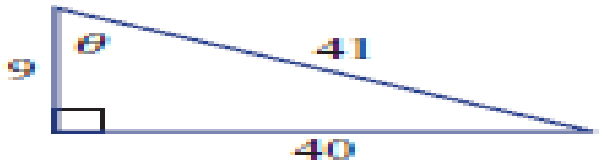
$\tan\theta =$

g



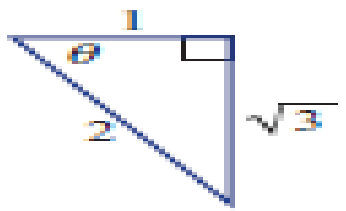
$\tan\theta =$

h



$\cos\theta =$

i



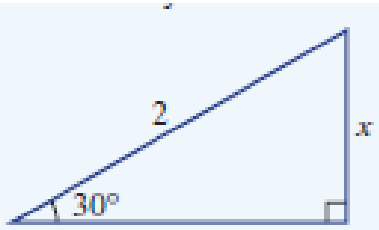
$\tan\theta =$

Lesson 6

Finding the Missing Side Using SOHCAHTOA

Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places where necessary.

a



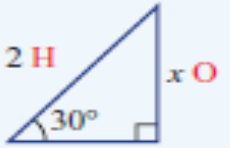
a $\sin\theta = \frac{O}{H}$

$\sin 30° = \frac{x}{2}$

$x = 2 \times \sin 30°$

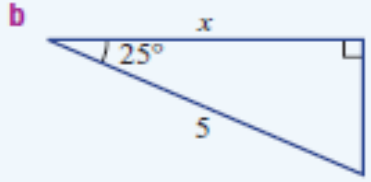
$\therefore x = 1$

Label the triangle and decide on your trigonometric ratio using SOH CAH TOA.
Write the ratio.



Solve the equation, using your calculator.

b



b $\cos\theta = \frac{A}{H}$

$\cos 25° = \frac{x}{5}$

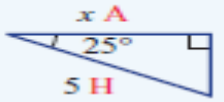
$x = 5 \times \cos 25°$

$x = 4.5315\ldots$

$\therefore x = 4.53$ (to 2 decimal places)

Label the triangle.
SOH CAH TOA
Write the ratio.

Solve the equation, using your calculator.
Round to 2 decimal places.



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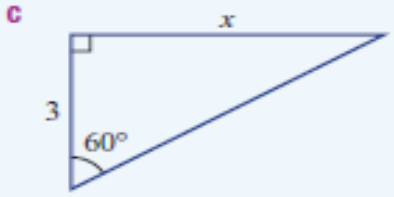


c $\tan\theta = \frac{O}{A}$

$\tan 60° = \frac{x}{3}$

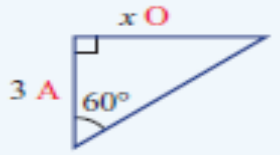
$x = 3 \times \tan 60°$

$x = 5.1961\ldots$

$\therefore x = 5.20$ (to 2 decimal places)

Label the triangle.
SOH CAH TOA
Write the ratio.

Solve the equation, using your calculator.
Round to 2 decimal places.



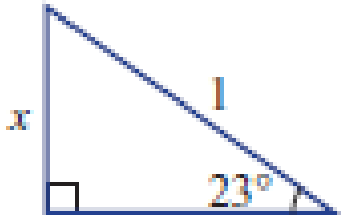
5 For the triangles given below:

i Copy each one and label the three sides opposite (O), adjacent (A) and hypotenuse (H).

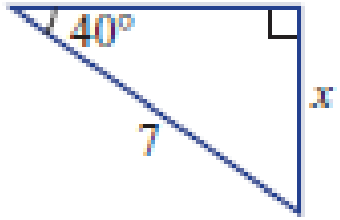
ii Decide on a trigonometric ratio.

iii Find the value of each pronumeral, correct to 2 decimal places.

a



b



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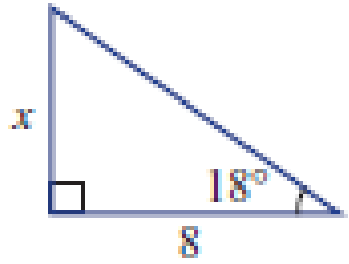
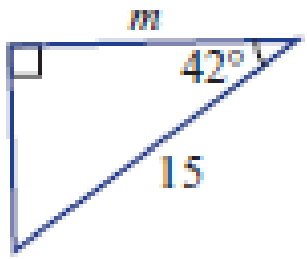
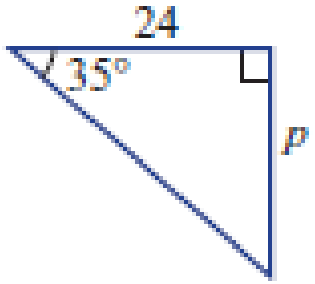


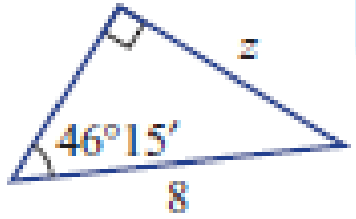
d



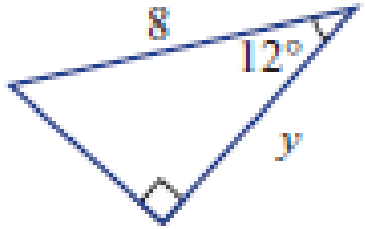
e



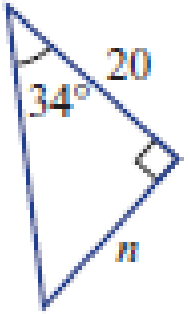
f



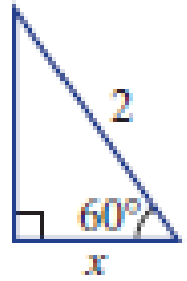
g



h

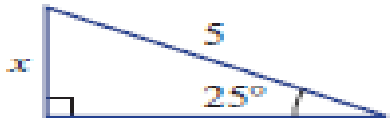


i

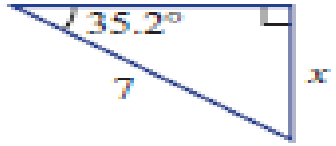


6 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.

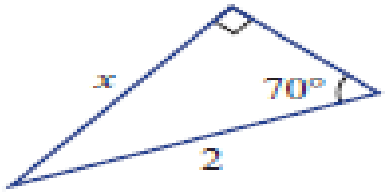
a



b



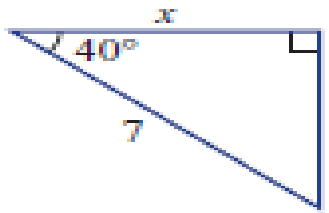
c



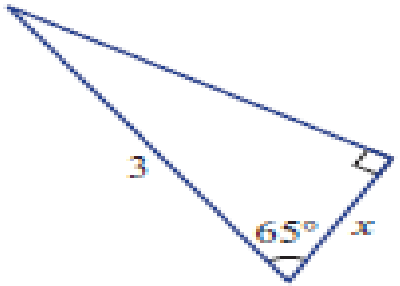
What ratio did you use for each of these?

7 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.

a



b



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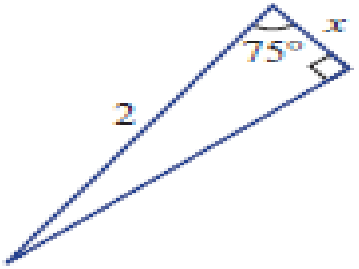
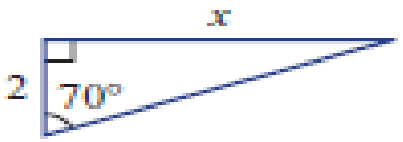


These three all use cos.

8 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.

a



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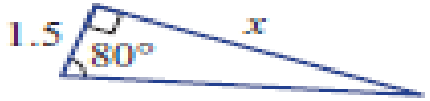

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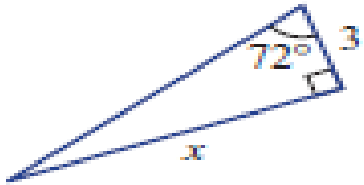
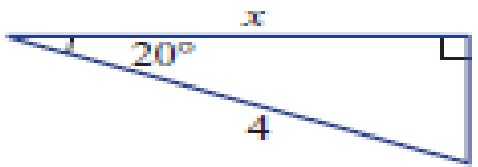


These all use tan.

9 Decide whether to use sin, cos or tan, then find the value of x in these triangles. Round to 2 decimal places.

a



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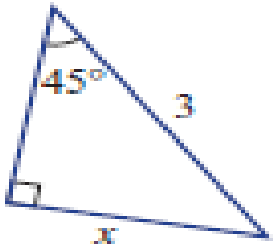

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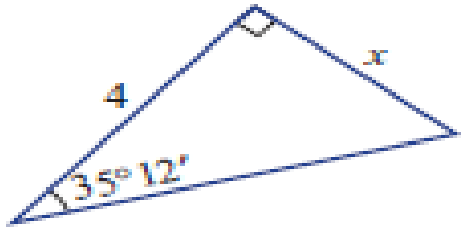


Solving for the Denominator

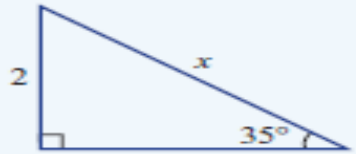
So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator and these problems can be solved with an extra step in your mathematical working.

https://www.youtube.com/watch?v=wGKwEPFXIHs

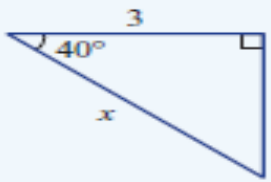
Example 12 Finding the value in the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

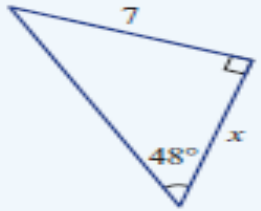
a



b



c

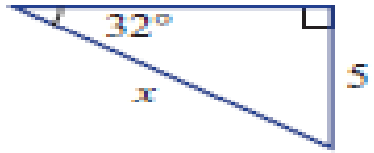


SOLUTION	EXPLANATION
a $\sin 35° = \frac{2}{x}$	Use $\sin\theta = \frac{O}{H}$, since we can use the opposite (2) and hypotenuse (x).
$x \times \sin 35° = 2$	Multiply both sides by x.
$x = \frac{2}{\sin 35°}$	Divide both sides by sin 35° to get x on its own.
$x = 3.48689\ldots$	Recall that sin 35° is just a number.
$\therefore x = 3.49$ (to 2 decimal places)	Evaluate and round your answer.
b $\cos 40° = \frac{3}{x}$	Use $\cos\theta = \frac{A}{H}$, as we can use the adjacent (3) and hypotenuse (x).
$x \times \cos 40° = 3$	Multiply both sides by x.
$x = \frac{3}{\cos 40°}$	Divide both sides by cos 40° to get x on its own.
$x = 3.9162\ldots$	Evaluate and round your answer.
$\therefore x = 3.92$ (to 2 decimal places)	
c $\tan 48° = \frac{7}{x}$	Use $\tan\theta = \frac{O}{A}$, as we can use the adjacent (x) and opposite (7).
$x \times \tan 48° = 7$	Multiply both sides by x.
$x = \frac{7}{\tan 48°}$	Divide both sides by tan 48° to get x on its own.
$x = 6.3028\ldots$	Evaluate and round your answer.
$\therefore x = 6.30$ (to 2 decimal places)	

4 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

a

b



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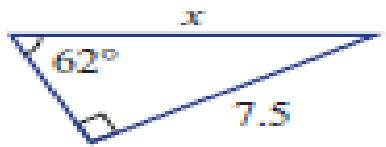
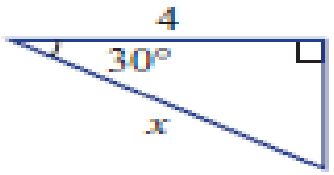


In $\sin 20 = \frac{3}{x}$, multiply both sides by x first.

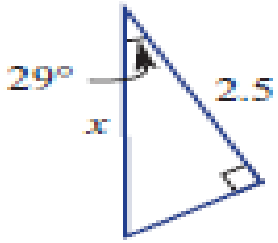
5 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

a



b

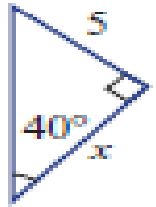
c



$\cos\theta = \frac{A}{H}$

6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

a



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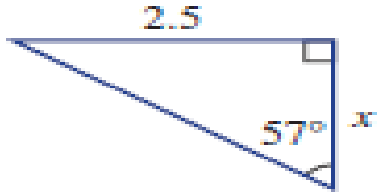

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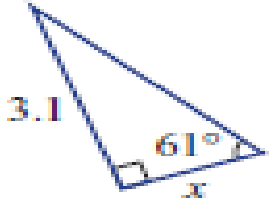
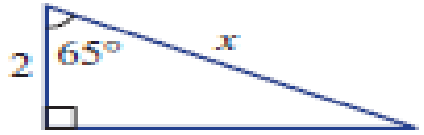


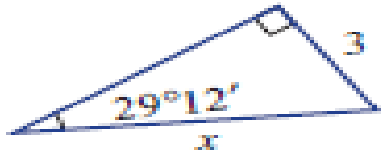
$\tan\theta = \frac{O}{A}$

7 By first deciding whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$, find the value of x in these triangles. Round your answer to 2 decimal places.

a



b



c

SOH CAH TOA